Fluid contacting a solid - Slip or no slip?

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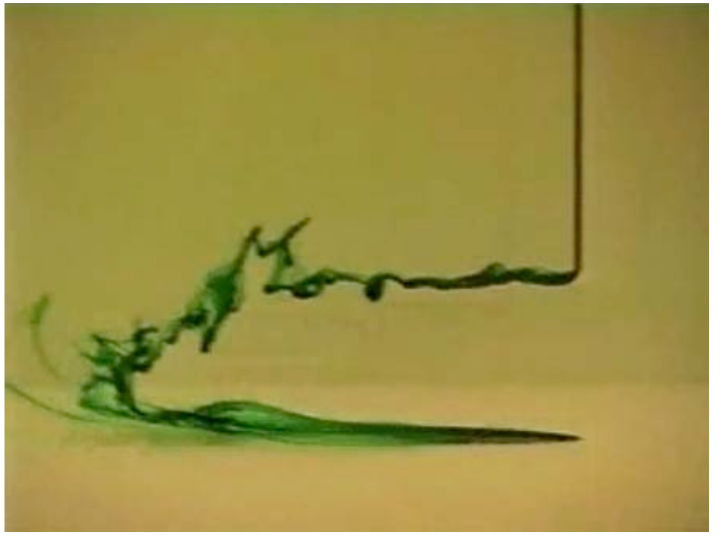
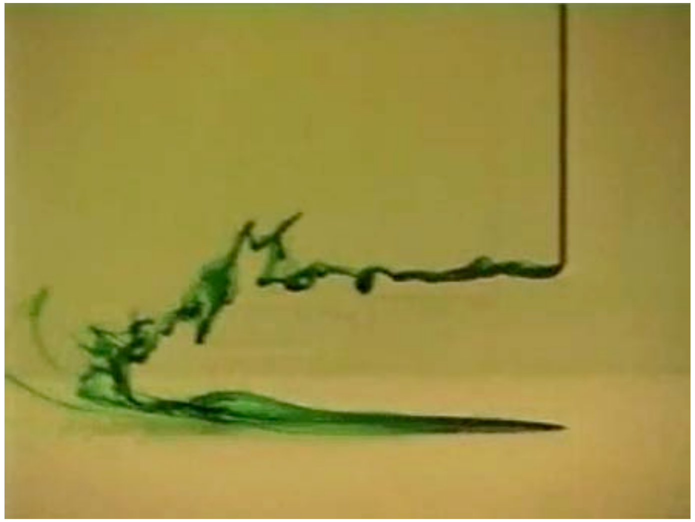
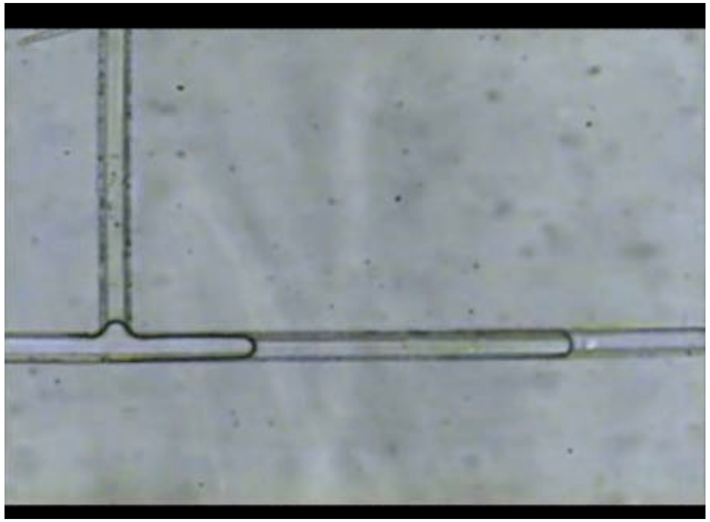


Slip vs No slip





- Hey it is so obvious!

- Eric Lauga 2005 : various phenomena in microchannels such as lower Re number for laminar-turbulent transition cannot be explained without slip
- Tretheway, 2002 : fluid velocity in hydrophobic channel is faster than that in hydrophilic channel
- Boundary between two liquids moves, which means that liquid near the triple phase contact line slips

Sydney Goldstein (From wikipedia and obituary on the Royal society)

- 1903 : Born in England
- 1916 : Became an orphan
- 1921 : Sunderland foundation allowed him to enter University of Leeds
  - Accommodated by Selig Brodetsky (Reader in Applied Mathematics)
  - Brodetsky found that Goldstein is a genius!
- 1922 : Brodetsky arranged him to transfer to St. John's College in Cambr

- **Career**
- Rockefeller Research Fellow, University of Göttingen, 1928-29.
- Lecturer in Mathematics, Manchester University, 1929-31
- Lecturer in Mathematics, University of Cambridge, 1931-45; Fellow of St. John's College, Cambridge, 1929-32, 1933-45 (Honorary Fellow, 1965)
- Spent 5 years to publish **'Modern developments in fluid dynamics'**
- Leverhulme Research Fellow, California Institute of Technology, 1938-39 (worked with von Karman)
- Back to England, working on aerodynamics for aircrafts, including supersonic ones 1939-45
- Beyer Professor of Applied Mathematics, Manchester University, 1945-50
- Professor of Applied Mathematics, 1950-55; and Chairman of the Aeronautical Engineering Department, 1950-54, Technion, Haifa, Israel, and Vice-President of the Technion, 1951-54 (Honorary Fellow, 1971)
- Gordon McKay Professor of Applied Mathematics at Harvard, 1954-68 then Professor Emeritus

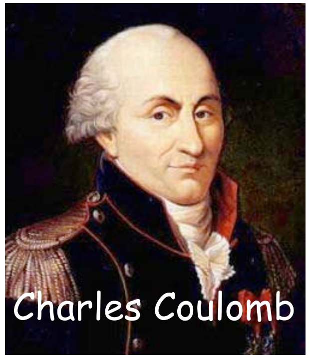
History regarding the no-slip phenomenon

Well, my famous Bernoulli theory does not match real measurements (Hydrodynamica,1738)

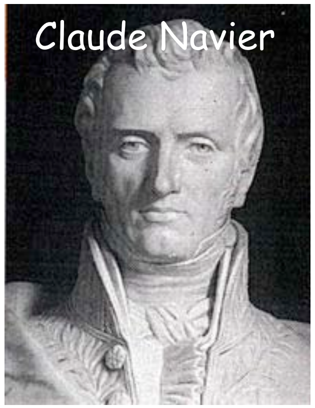
Pierre De Buat

If the mean velocity is very slow, the fluid is at rest at the proximity to the surface (Principes d'Hydraulique,1786)

Then there should be a thin layer which is 'stuck' to the surface, and bulk fluid might slip on it (1813~15)



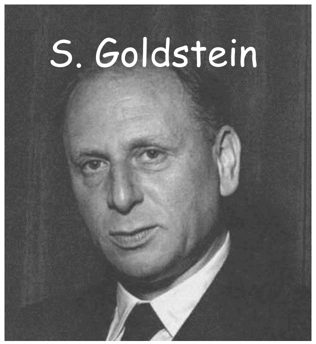
Du Buat's right. My tests showed that the surface property does not affect the drag force (1800)



It may or may not slip, depending on the fluid and surface property, depending on the extrapolation length(Ibid, 1823)

Poiseuille, Hagen, Maxwell, Couette

Now we can see with microscope. Fluid does not seem to slip (1841 ~ 1890)



'No slip' is widely accepted now, even though there are extreme situations when the fluids slip like Maxwell's rarefied gas (1938)

Modern developments in fluid dynamics

- This book is cited by anybody who wants to debate about slip phenomenon, but the actual paper in the book is just a review

- Goldstein's other papers in the book are
  - Drag of a disk in a fluid
  - Vorticity transport theory
  - Pressures in a turbulent stream
  - Stability of viscous fluid flow under pressure
  - Boundary layer growth
  - a note on roughness
  - Similarity theory

  - And he says

    the condition of no slip at the boundary is then applied to complete the solution.

    regarding the slip condition in those papers

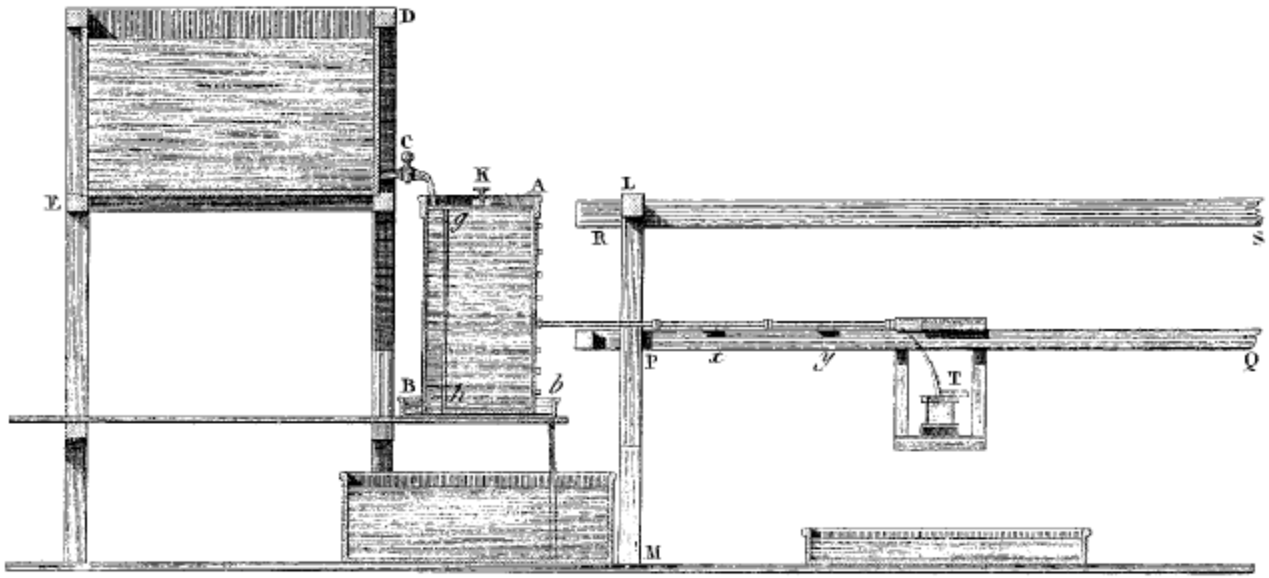
before Navier, Girard's capillary tubes

- Girard believed that there should be a thin adherent layer on the surface, and the bulk liquid slips on it. He also assumed the thickness of the layer would be dependent on the adhesion force between the fluid and solid

- To prove his claim, he used a) copper tube b) glass tube to see the difference. It seems that he didn't measure the diameter himself and just believed the numbers offered by the manufacturer

- He actually got 'difference'! However, same tests done later by Gotthilf Hagen didn't show any difference.

- Navier believed Girard'
  test result



Navier equation

- Navier has used Laplace's theory on elasticity for his boundary condition development

$$\delta \mathbf{R}_{\alpha\beta} = \mathbf{u}(\mathbf{r}_\beta) - \mathbf{u}(\mathbf{r}_\alpha)$$

$\mathbf{w}(\mathbf{r})$= a virtual displacement of the particles of the solid

$$M = -\frac{1}{2} \sum_{\alpha\beta} \phi(R) u_R w_R.$$

$\phi(R_{\alpha\beta})$ the proportionality coefficient being a rapidly decreasing function

$$M' = -\frac{1}{2} \sum_{\alpha\beta} \psi v_R w_R$$

$$\oint \varepsilon(\partial_i v_j + \partial_j v_i) w_i dS_j$$

$$\varepsilon = \frac{2\pi}{15} \int N^2 \psi(R) R^4 dR.$$

$$E\mathbf{v} + \varepsilon \partial_\perp \mathbf{v}_{//} = \mathbf{0},$$

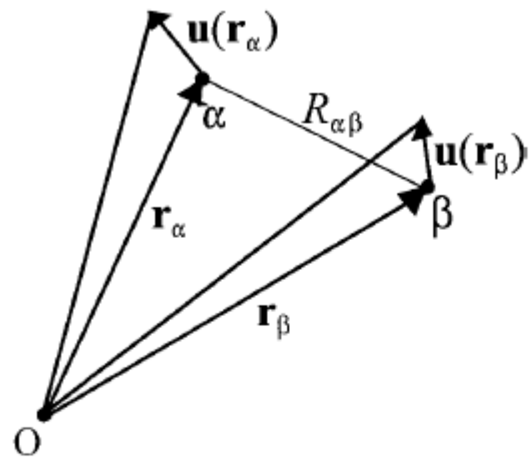


Diagram for displacements in an elastic body

Navier's extrapolation length

- Shear stress will have only 'velocity gradient ' in the fluid, but the same stress can incur slip at the solid surface

For solid

$$\sigma = k v_s$$

For fluid

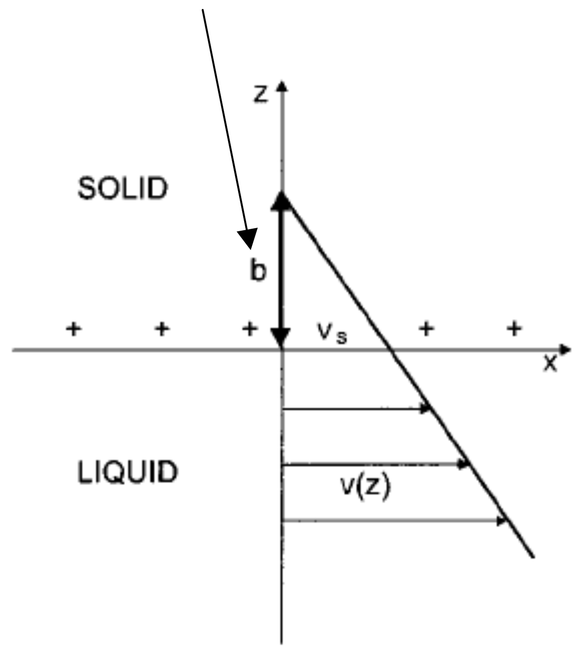
$$\sigma = \eta \left| \frac{dv(z)}{dz} \right|$$

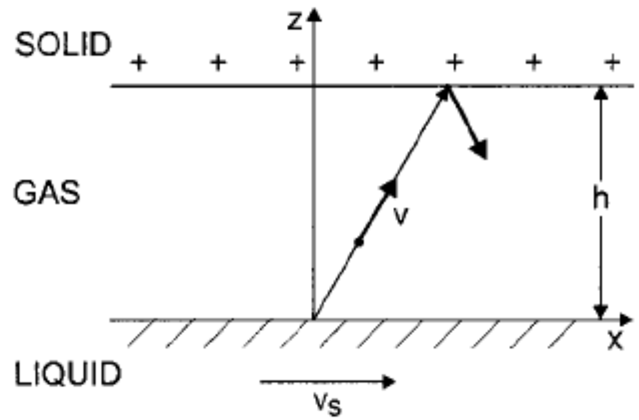
Equal stress gives

$$b = \eta / k$$

- For normal Newtonian fluids, this b is on the order of the molecule size, leading to no slip condition
- For polymer melts against surface with grafted chains, b jumps to μm range
- If there is any very thin gaseous layer (thinner than mean free path), then b is independent of the thickness of the layer (because this layer will be in a condition similar to rarefied gas) and again jumps to μ m range

Navier's extrapolation length





Extrapolation length – polymer melts (deGennes)

- Before surface grafting
  - $k = k_m$ (friction coefficient for monomer)
  - b ~ 100μm (easily viewed during extrusion process)

- After grafting, low shear stress

$$b_0 \cong (\nu R_0)^{-1}$$

where $R_0 = Z^{1/2}a$ is the coil size of the grafted chains (Z is the number of monomers per grafted chain and a is a monomer size).

- After grafting, high shear stress
  - coils of the grafted chains start to stretch (assuming different grafted chains do not overlap), leading to Rouse friction

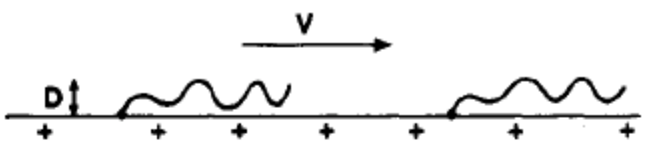
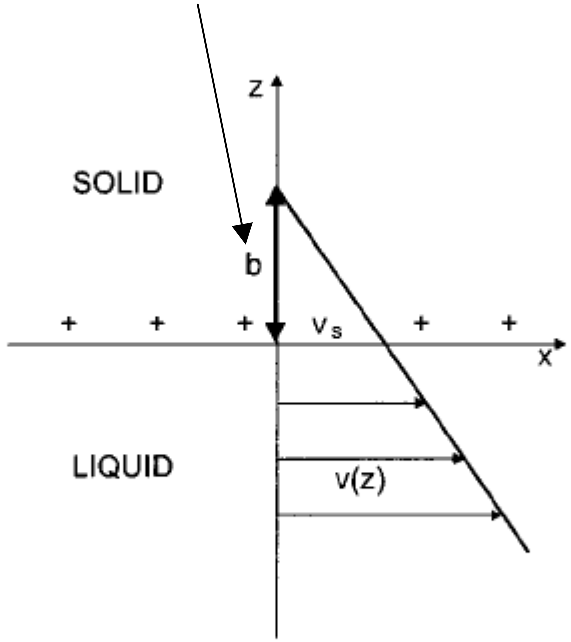


Figure 3. A grafted chain under shear flow in a melt. Where the shear stress σ exceeds a low threshold σ^* (eq 22), the chain is strongly stretched.

Navier's extrapolation length



Maxwell's slip for rarefied gas

- This is a part of his paper on the viscosity in rarefied gas (1879)

- Rarefied gas is not continuum, so

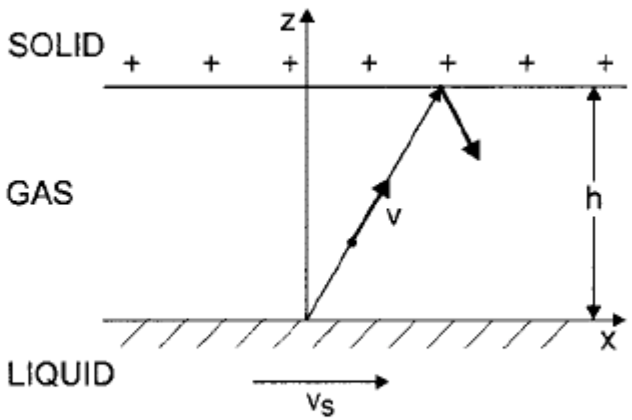
$$\vec{u}_{\text{slip}} = -\frac{(2-\sigma)}{\sigma\mu}\lambda\vec{\tau} - \frac{3}{4}\frac{N_{\text{Pr}}(\gamma-1)}{\gamma p}\vec{q}, \quad \vec{\tau}=(\vec{i}_n\cdot\mathbf{\Pi})\cdot(1-\vec{i}_n\vec{i}_n),\ \vec{q}=\vec{Q}\cdot(1-\vec{i}_n\vec{i}_n)$$

$$u_s = \frac{(2-\sigma)}{\sigma}\lambda\frac{\partial u_x}{\partial n} + \frac{3}{4}\frac{\mu}{\rho T}\frac{\partial T}{\partial x} \quad \text{(1D case)}$$

σ : the momentum accommodation coefficient

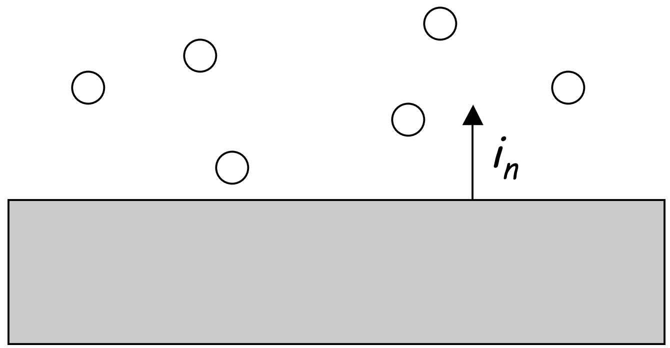
m : gas viscosity

λ : mean free path



wall

the wall



Role of this rarefied gas (deGennes)

- When a thin air layer exists between liquid & solid, it acts like a rarefied gas

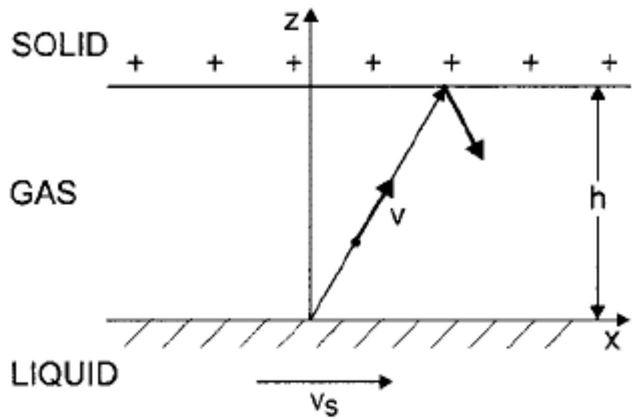
$$\sigma = m v_s \frac{\rho}{m} \bar{v}_z = \rho v_s v_z$$

$$\bar{v}_z = \int_0^\infty \frac{1}{(2\pi)^{1/2} v_{th}} v_z e^{-v_z^2/2v_{th}^2} \, dv_z = v_{th}/(2\pi)^{1/2}$$

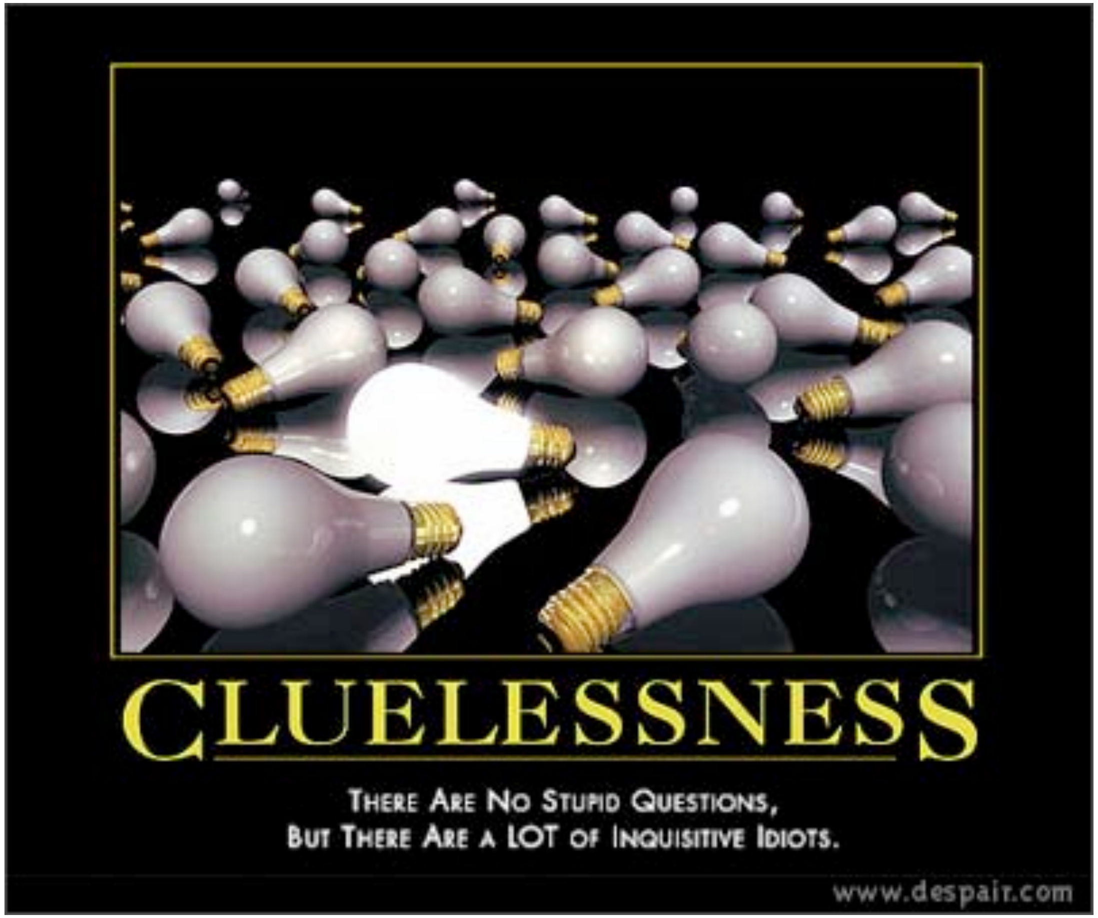
$$v_{th}^2 = kT/m.$$

$$b = -h + \frac{\eta}{\rho \bar{v}_x} \cong \frac{\eta}{\rho v_z} \qquad (h < l)$$

If we choose typical values, $\rho = 1$ g/L, $v_{th} = 300$ m/s, and $\eta = 10^{-2}$ P s, we find $b = 7\ \mu$m.



Thanks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